

Statistics 624: Statistical Computing
Homework # 12
Due Thursday, 1 Nov., 8am

In this assignment, please complete, organize, and turn in **on paper** (just had to see if you were still paying attention to my notes up here) to `stat624@stat.byu.edu`.

This assignment demonstrates root finding using Newton Raphson (in one dimension) .

Estimation of the size of a closed population (N) is an important consideration in wildlife biology. The approach used in many estimation problems is that of capture/recapture. Capture/recapture methods can generally be characterized as repeatedly sampling (t times) from a closed (fixed N) population and marking the captured animals with a tag. When the animals are captured again, they can easily be distinguished from the “captures” by the tag and they are called “recaptures”. There are several models for capture/recapture. One such model was proposed by Darroch (1958). In his approach, he finds that the MLE for N , \widehat{N} , is the solution to

$$\left(1 - \frac{M_{t+1}}{N}\right) = \prod_{i=1}^t \left(1 - \frac{n_i}{N}\right),$$

where M_{t+1} represents the number of unique animals captured, n_i is the number of animals sampled in the i^{th} sample, and t is the number of samples.

A wildlife biologist conducted 4 samples of various sizes from a Bowhead whale population and captured 79 unique whales. The four sample sizes were $n_1 = 30$, $n_2 = 15$, $n_3 = 22$, $n_4 = 45$. Write a program to compute the maximum likelihood estimator for N , the Bowhead whale population size.

In your program, make sure you do the following:

- Compute the MLE to precision 10^{-14} .
- Report \widehat{N} and the number of iterations.
- Make sure you don't get into an infinite loop by never reaching that precision, that is, use a `maxiter` variable.
- Compare your results with that of the built-in R function `uniroot`. Report the number of iterations required for your N-R code versus `uniroot`.

Darroch, J.N. (1958) “The Multiple-Recapture Census: I. Estimation of a Closed Population,” *Biometrika*, **45**, 343–359.