Statistics 624: Statistical Computing
Homework #14
Due Friday, 16 Nov., 9 am

In this assignment, please complete, organize, and turn in **electronically** to `stat624@stat.byu.edu`.

1. Let μ denote a vector of length p and Σ denote a positive-definite $p \times p$ matrix. If $\mathbf{z}$ is multivariate normal with mean vector $\mathbf{0}$ and covariance matrix $\mathbf{I}$, prove that $\mathbf{x} = \mu + \mathbf{Tz}$ is multivariate normal with mean vector μ and covariance matrix Σ if $\mathbf{TT}' = \Sigma$.

2. Write code to simulate from a multivariate normal with

$$\mu = \begin{bmatrix} -3 \\ 0 \\ 5 \end{bmatrix} \qquad \text{and} \qquad \Sigma = \begin{bmatrix} 10 & -8 & 5 \\ -8 & 15 & 6 \\ 5 & 6 & 20 \end{bmatrix}$$

   Confirm your results by taking a sample of 500 random vectors and constructing the sample mean vector and sample covariance matrix.

3. One model for a randomized block design is to organize the data y_{ij}, the measurement from treatment i and block j, into

$$\mathbf{y} = \begin{bmatrix} y_{11} \\ y_{12} \\ \vdots \\ y_{1b} \\ y_{21} \\ y_{22} \\ \vdots \\ y_{2b} \\ \\ \vdots \\ \\ y_{t1} \\ y_{t2} \\ \vdots \\ y_{tb} \end{bmatrix}$$

Differences in the mean between treatments would be reflected in different values of

$$\mu_i = \mathrm{E}(y_{ij}), \qquad j = 1, \ldots, b.$$

The *block effect* is represented by covariance within a block. That is, the covariance matrix of $\mathbf{y}$ is

$$\mathrm{V}(y_{ij}) = \phi_0 + \phi_1 \quad \text{and} \quad \mathrm{Cov}(y_{ij}, y_{i'j'}) = \begin{cases} 0 & \text{if } j \neq j' \\ \phi_1 & \text{if } j = j' \end{cases}$$

That is, observations in the same block are correlated, but observations in different blocks are independent.

(a) For a randomized complete block design with 3 treatments and 4 blocks, write μ and Σ.

(b) Generate the data for an randomized complete block experiment where $\mu_1 = 0, \mu_2 = 6, \mu_3 = 4$ and $\phi_0 = 6, \phi_1 = 2$.