Recent research has indicated that mental health issues are on the rise at US Universities. As a result, many Universities offer free or reduced cost mental health services. The website http://madison.byu.edu/bayes/counseling.dat represents a sample student counseling sessions with therapists. The primary interest is in the improvement in OQ45 scores from starting levels. We would like to determine the extent of improvement from baseline for each student. Note that each student (patient) has a potentially different number of session. We would like to know the mean improvement for each student and then also estimate the mean improvement across all students. That is, assume $Y_{ij} \sim N(\theta_i, \sigma^2)$ and fit a hierarchical model, where $i = 1, \ldots, m$ indexes student and $j = 1, \ldots, n_i$ indexes the visit for student $i$.

- Choose a prior distribution for the parameters of the likelihood function. Justify this prior distribution!

- Use MCMC techniques to:

  1. Calculate the posterior distribution of the parameters of the model (all of them).
  2. Calculate $E[\Theta|Y]$, the posterior mean (the whole thing).
  3. Calculate $V[\Theta|Y]$, the poserior variance (how are you going to summarize this???).
  4. Calculate the posterior predictive distribution of the “next” average improvement score $(\theta_{m+1})$.
  5. In general, a change of 15 points on the OQ45 scale is considered “clinically meaningful”. Calculate the probability that a randomly selected student experiences a clinically meaningful change in mental health status.
  6. Calculate the probability that the mean improvement across all students exhibits clinically meaningful change in mental health status.
• Graphically show the shrinkage effect of the hierarchical structure of the model. That is, on one horizontal line, show the 30 MLE’s of the 30 patients. On another horizontal line, show the posterior means of the 30 patients. Connect with lines the MLE for each patient to the posterior mean for each patient. Please discuss the amount of shrinkage for several points and whether you “believe” that is an appropriate amount of shrinkage.