1. Let $\mu$ denote a vector of length $p$ and $\Sigma$ denote a positive-definite $p \times p$ matrix. If $z$ is multivariate normal with mean vector $0$ and covariance matrix $I$, prove that $x = \mu + Tz$ is multivariate normal with mean vector $\mu$ and covariance matrix $\Sigma$ if $TT' = \Sigma$.

2. Write code to simulate from a multivariate normal with $\mu = \begin{bmatrix} -3 \\ 0 \\ 5 \end{bmatrix}$ and $\Sigma = \begin{bmatrix} 10 & -8 & 5 \\ -8 & 15 & 6 \\ 5 & 6 & 20 \end{bmatrix}$.

Confirm your results by taking a sample of 500 random vectors and constructing the sample mean vector and sample covariance matrix.

3. One model for a randomized block design is to organize the data $y_{ij}$, the measurement from treatment $i$ and block $j$, into

$$
\mathbf{y} = \begin{bmatrix} y_{11} \\ y_{12} \\ \vdots \\ y_{1b} \\ y_{21} \\ y_{22} \\ \vdots \\ y_{2b} \\ \vdots \\ y_{t1} \\ y_{t2} \\ \vdots \\ y_{tb} \end{bmatrix}
$$
Differences in the mean between treatments would be reflected in different values of

\[ \mu_i = \mathbb{E}(y_{ij}), \quad j = 1, \ldots, b. \]

The block effect is represented by covariance within a block. That is, the covariance matrix of \( y \) is

\[
V(y_{ij}) = \phi_0 + \phi_1 \quad \text{and} \quad \text{Cov}(y_{ij}, y_{i'j'}) = \begin{cases} 
0 & \text{if } j \neq j' \\
\phi_1 & \text{if } j = j'
\end{cases}
\]

That is, observations in the same block are correlated, but observations in different blocks are independent.

(a) For a randomized complete block design with 3 treatments and 4 blocks, write \( \mu \) and \( \Sigma \).

(b) Generate the data for an randomized complete block experiment where \( \mu_1 = 0, \mu_2 = 6, \mu_3 = 4 \) and \( \phi_0 = 6, \phi_1 = 2 \).